A Response to the 1998 Discussion Draft of the NCTM Principles and Standards for School Mathematics

compiled and edited by

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April 30, 1999

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The Access and Assets Project is a collaborative effort of the Education Development Center (EDC) and the American Association for the Advancement of Science (AAAS) to promote high quality, standards-based education for students with disabilities. One of the major activities of this project is to provide information to standards committees and other educational organizations about how national standards and policies can be crafted and implemented to insure the inclusion of students with disabilities in standards-based science and math education and to develop a model of inclusion that is based on an asset rather than deficit perspective on disability.

As part of this project we have conducted a review of the 1998 NCTM Principles and Standards Draft. The review was encouraged by Joan Ferrini Mundy, Chair of the NCTM Standards 2000 Writing Group and Douglas Clements, Member of the Grades Pre-K - 2 Writing Group, who both serve on the Advisory Board of the Assets and Access Project. The review was performed by project staff and consultants in collaboration with members of our advisory board and colleagues from the field. The review team (a list of the reviewers is included in the Appendix) consisted of math and science educators, scientists, disability advocates, and researchers, and included individuals with disabilities and parents of children with disabilities. Reviewers were invited to submit comments on selected sections of the draft. In particular, we asked them to consider what kinds of change may be need in the draft in order to

- Recognize and utilize the assets that students with disabilities bring to learning math; and
- Insure that the standards will be inclusive for children with all types of disabilities (e.g., physical, sensory, cognitive, developmental, emotional, learning disabilities)

This report summarizes and synthesizes individual reviewer's comments. It was prepared by project staff and reviewed by all of the reviewers who submitted comments. The report is organized in two main sections. The first section includes general comments and the second section contains detailed suggestions for revision of Chapters 1 and 2. The general comments section presents the overall themes from which the specific comments were derived.

This report was prepared based on an asset model of disability and emphasizes the inclusion of all students with disabilities in standards-based education. The asset model implies that the experience of disability is seen to inform mathematical and scientific understanding instead of being viewed as a deficit that must be compensated. We believe that a focus on assets is important because students with disabilities bring unique skills and experiences relevant to mathematics and science and that a focus on these assets can enhance their achievement of the standards and contribute to a more inclusive formulation of the standards themselves (see Appendix B for an essay by Harilyn Rousso, Project Consultant and one of the reviewers, that describes the asset model in more detail).

We also believe that it is important that the standards are inclusive of all students with disabilities. Implicit in data collection practices and explicit in the tracking systems is the belief that some students simply cannot learn some kinds of material because of their disability. It may be justifiable to conclude that students with mental retardation or other types of developmental disabilities may have different capacities for grasping higher level cognitive tasks; however, it is more often low expectations and entrenched notions rather than accurate assessments that limit what students are taught and expected to know. This is particularly true for children with physical and sensory impairments whose intelligence falls in the "normal" range, but who are nevertheless often excluded from educational opportunities and from data collection about participation and achievement.

The scope of our comments and suggestions is limited in several ways and for several reasons:

Given the short time frame the project had available for the review and the length of the Principles and Standards Draft, we were not able to review every chapter. Since the focus of the Asset and Access Project is on the early elementary grades, and many of the colleagues who are collaborating with us bring expertise in early childhood education, we decided to focus our review efforts on Chapter 2 (Guiding Principles for School Mathematics Programs), Chapter 3 (Overview of Standards for Grades PreK-12) and Chapters 4 and 5 which present the Standards for Grades Pre-K to 2 and 3 to 5 respectively. The general comments section presents the major themes that have emerged from these reviews. Specific suggestions for revision are for Chapter 1 (Introduction), and Chapter 2 are included also, and a set of comments about Chapters 3 and 5. Our comments and suggestions are meant to exemplify how other sections may need to be revised.

An important aspect of fostering the inclusion of students with disabilities in standards-based education would be to add concrete examples of curriculum activities and units that have proven to be successful in enabling students with and without disabilities to achieve standards-based learning outcomes in math. While we have some preliminary ideas for such examples, they have not been rigorously tested in a variety of classrooms, and therefore we feel that at this point it would be premature to suggest them for inclusion into the draft. One of our goals in the Assets and Access Project is to develop, implement and evaluate such examples. While it may not be possible to include these and other examples into the printed version of the Principles and Standards Document because of the schedule for revision and production of the final draft, it would be useful if the opportunity of adding them to the electronic version of the Principles and Standards Document at a later point in time could be preserved.

One difficulty we encountered in organizing this review was to obtain a version of the Principles and Standards Draft that was accessible to blind reviewers, which considerably cut down on the time they had available for the review. Despite the support from NCTM staff and the availability of electronic versions of the Draft, it took several weeks and numerous hours of work to get just one chapter formatted in such way that it was readable by our blind reviewers. The electronic version of the draft that was available in PDF format on the Internet was not useable because it was edit-protected and therefore could not be copied into other programs that included text-to-speech capabilities. A rich-text version of the draft that we received on disk from NCTM about 2 weeks after we requested it was not compatible with the text-to-speech program that one of our reviewers used. We finally ended up working with the electronic version that is available in html format on the Internet. We were able to copy the text from this version into a word processing program, but careful line-by-line editing was required by both our staff and the reviewer to eliminate unreadable characters and to correct formatting. NCTM needs to insure that the final draft of the Principles and Standards document will be accessible to people with disabilities by making it available in multiple formats.

General Comments

Many of the reviewers commented that, overall, the draft was quite good. In particular, the reviewers welcomed the elaboration of the equity principle and the centrality that this principle is given in the current draft compared to the 1989 Standards document. All of the reviewers, however, expressed strong **concern over the noticeable absence of disability in the definition of “all students” and the lack of acknowledgment that students with disabilities have traditionally been underrepresented in mathematical careers**

and programs (c.f., National Science Foundation, 1996). Many of our specific suggestions for revision thus consist of adding “disability” to the definition of “all students.”

The reviewers also had a very positive reaction to the emphasis on promoting high expectations for all students, and the recognition of the need for raising expectations for traditionally underrepresented groups. Again, however, the reviewers emphasized that **students with disabilities need to be explicitly mentioned as a group of students for whom expectations need to be raised.** Low expectations have traditionally been one of the major barriers that have kept students with disabilities from participation and achievement especially in math. Because other skills such as reading and writing have often been thought to be more important than math and science, many children with disabilities have not had many opportunities to participate in math and science classes. And even if they do participate, children with disabilities are often not expected to perform at the same level as their nondisabled peers because disability has been viewed predominantly as a deficit. For instance, students with disabilities are often pushed to develop procedural proficiency rather than to gain conceptual understanding, either because they are perceived as not needing to understand math for their futures, or because they are perceived as incapable of deeper understanding. By emphasizing the assets that *all* students bring to the math classroom, and providing the flexibility for nontraditional approaches to doing math, NCTM could make an important contribution to raising expectations for students with disabilities.

Embed the NCTM recommendations about including students with disabilities in the context of other relevant policy and law. The Principles and Standards draft, with its emphasis on promoting high standards for all students, is consistent with other standards and legislation that affect students with disabilities, such as the Americans with Disabilities Act (ADA), the Individuals with Disabilities Education Act (IDEA), Goals 2000, Improving America’s Schools Act (IASA), and the National Science Education Standards (NRC, 1996). Both Goals 2000 and Improving America’s Schools Act are explicit that students with disabilities and diverse learning needs are part of the definition of “all students.” Under the newly-reformulated IDEA, *students with disabilities must, for the first time, be included in all standardized assessments*; Goals 2000 and Title I of the IASA require states to provide for participation of students with diverse learning needs, including providing accommodations and adaptations to permit their participation. These students’ results must be considered in the planning of curriculum by the school districts, and in reports made to the state. It is therefore essential that all efforts are made to ensure that students with disabilities have the opportunity to master the same curriculum as their counterparts. The revision writers may want to make specific mention of the mandates in these laws; the 1996 NRC *Educating One and All* does a nice job of this.

A number of the ideas delineated in the document hold particularly true for students with disabilities, and we highlight them here, along with comments about the way in which the emphasis might be strengthened to serve students with disabilities:

- High expectations and their need to be coupled with strong support for students that will enable them to accomplish the desired learning outcomes is nicely addressed in Chapter 2. Expanding on that throughout the document would help students with disabilities, if you can add something that reflects the idea that disability implies a different way of performing some activities, but that no student is handicapped until we create barriers to the different ways that that student learns. Or said more positively, each of us learns in many different ways, and it is our job as educators to figure out how to support that learning in our students.

- The recognition that mathematics content needs to be useful and connected to students' prior knowledge, experience and interest, will serve all students, and especially students with disabilities. As one of our reviewers put it, "as a mother of a child with a developmental disability, I know that the only math my daughter can understand is math that in some way relates to her world; abstract concepts are incomprehensible for her." A teacher reviewer noted that

  > In my experience, disabled students fare better by far if they are given real problems which they must solve for a purpose. Children who struggle day after day with pages of algorithmic problems quickly give up. However, when the problems become embedded in a real-life context, when the children are given the authority to figure out the best and most expedient way to solve the problems, when they are given the tools to work out the issues, they rise to the occasion. Keeping journals of their experiments, developing their own language and then learning to translate that into the more common language of mathematics is the key.

- Regarding active engagement with math through various means, including manipulatives, technology, personal involvement, or collaborative problem-solving: One common misconception of students with physical, sensory, or attentional disabilities is that they cannot handle manipulatives or technology as they might "break " the equipment, or might be too "distracted" by it.

- Access to technology, mentioned in the Technology principle, is critical for students with disabilities. To deny students with disabilities equitable access to technology in the learning of mathematics is to "handicap them, and that is fundamentally inequitable" as is stated so clearly in the Discussion Draft. It would be important to refer as well to assistive technology, that is, technology designed to enhance participation and capacity, and to make mention of devices which can further understanding of mathematical principles by students without as well as with disabilities. As one of the reviewers said,

  > Technology "has enabled students with disabilities to show more readily than they otherwise could what they really know. In addition to the calculator, the computer is an essential tool which can be modified to serve the varying needs of disabled youngsters...They are also tools which faciliate problem solving as they can be used to help organize and systematize data, picture data in several representational forms, and consistently apply operational rules to the data...[This frees the student] to use his/her cognitive energies for the task rather than the myriad processes within the task."

It is important that **the commitment to include students with disabilities in the definition of "all students" be reflected not only in the text, but also in the examples** that are included in the printed and electronic versions. Examples could consist of: curriculum activities that build on the strength of students with disabilities, description of how such activities were developed, stories about mathematicians with disabilities, images of students with disabilities participating in math learning, etc.

Describe students as the heterogeneous group they are. It is important to recognize that there is heterogeneity among *all* learners. The way in which the phrase "all students" is used throughout the document implies that "all children" are essentially the same in development, experience and prior knowledge, and the ways in which they learn most effectively and easily. If instead the document can acknowledge the range, variation, and diversity among "all children," then it becomes easier to talk about students with disabilities in concrete terms without their being the only special group singled out for attention and assistance. Incorporating phrases or qualifiers throughout the text that indicate “some children might have more or less difficulty with this concept depending on their level of experience or development” might help. Then there can be specific reference to disability as well, about how to make a concept more accessible, both physically and conceptually, for example, to a student who is blind or deaf, autistic or mentally retarded.

It is also important to recognize that a number of different disability types are covered by the phrase “students with disabilities.” More than 5 million students, about 10 percent of the school-age population, have disabilities and qualify for special education services. While children with disabilities have things in common, among them that their education is governed by a set of policies detailed in the Individuals with Disabilities Education Act (IDEA), it is important to keep in mind that they are also quite a heterogeneous group. Figure 1 shows the number and percent of children in the U.S., by disability type, who receive special education services. Each type of disability brings its unique set of strengths in relation to mathematics. Even within a particular disability group there can be a large degree of variation in the kinds of mathematical talents an individual has.

Figure 1: Number of Children Ages 6-21 Served Under IDEA, Part B During the 1995-96 School Year

Type of Disability	Total	Percent
All Disabilities	5,070,658	100%
Specific Learning Disabilities	2,597,231	51%
Speech or Language Impairments	1,025,941	20%
Mental Retardation	585,308	12%
Serious Emotional Disturbance	438,217	9%
Multiple Disabilities	94,156	2%
Hearing Impairments	68,070	1%
Orthopedic Impairments	63,200	1%
Other Health Impairments	133,419	3%
Visual Impairments	25,484	0.5%

Autism	28,827	1%
Deaf-Blindness	1,362	0.02%
Traumatic Brain Injury	9,443	0.2%

Source: U.S. Department of Education (1997). Nineteenth Annual Report to Congress on the Implementation of the Individuals with Disabilities Education Act. Washington, DC: U.S. Government Printing Office

Describe the consequent need for varied approaches to teaching and assessment. Equally important is that there be explicit mention of the need to provide a diversity of teaching strategies, and multiple formats and multisensory presentations of materials. We would also urge that there be repeated references to multiple paths to solving problems, which goes along with the ideas of students inventing and describing their own strategies and algorithms. It may come in under the teaching principle, or in examples throughout, and it doesn't necessarily have to refer to students with disabilities. The point to make is that students may have a variety of ways of approaching a particular mathematical problem, some more elegant and efficient than others, some more inventive and creative. Judgment of their relative utility then needs to connected to a set of criteria around accuracy of results, understanding, and mastery.

Then there are the conundrums. We have identified a few based on our reviewers comments, including:

1. Curricular sequence. Our reviewers' comments pose an interesting challenge to a standards document that is organized by grade level and an assumption of a certain linearity. On the one hand, said one teacher reviewer,

> The most common concern articulated by special education teachers working with students in grades 3-5 in mathematics is their recognition that many of their students have not achieved mastery of even the most basic skills and concepts contained with the pre-k to grade 2 sequence. As a result they feel both threatened and dishonest when they are required to build upon skills and concepts which have not been mastered. As one teacher put it, "I am being asked to build a house of brick on a foundation of sand."

On the other hand, a parent and educator made it very clear (and her full remark is detailed later) that it was only through the intervention of a creative teacher that her autistic daughter was freed from "calculation purgatory" to which she had been consigned year after year because she couldn't grasp basic number operations. For her daughter, changing the order, in this case tackling fractions, opened up mathematics. Thus while NCTM's view that mathematical concepts need to build on each other is well taken, decisions about sequence may need to be presented as more malleable. As the reviewer said, "For children with disabilities...their unique world view may make traditional 'linear' views of the order of math education inappropriate." It may be advisable to include some mention of the tension between mastering skills and moving on even if the skills haven't been mastered but revisiting them later when other experiences and learnings are in place.

2. Time. Students with disabilities – and often students without disabilities – may need more time to accomplish given tasks, participate in assessments, and acquire skills and concepts. The conundrum here is how to determine when additional time is a support, and when it is a sign of lowered expectations that results in less knowledge gained. It may be useful for the Principles and Standards document to highlight this concern, and to ensure that students be given the "gift of time so that they can process at a pace which is reasonable for them." This would seem to apply equally well to students without disabilities.

3. In order for teachers to be able to include students with disabilities fully in standards-based reform requires a level of support and knowledge that do not currently exist. This could probably be said as well in relation to students without disabilities, but the fact is, we don't have much in the way of research-based strategies or tested curricular examples yet. Research on teaching strategies has produced some useful results (see, for example, the Department of Education descriptions in their Annual Report to Congress on the Implementation of IDEA on research about such teaching strategies as Explicit Instruction, Anchored Instruction, and Cognitive Strategy Instruction); but this work, while promising, is in fairly early stages and not well disseminated. There is a terrific need for professional development, and again, although there are promising strategies like collaborative teaching pairing math specialists with special and general educators, there is relatively little available on a regular basis for the average teacher who wants to ensure the inclusion and achievement of students with disabilities in mathematics.

One of the toughest issues in professional development is how much teachers need to know about individual disabilities, how to analyze where students need extra assistance and accommodation because of their disability, and how not to focus on the disability but on the contributions to the learning of mathematics based both on the unique experience of disability and on the particular talents and intelligence that the individual brings.

It seems important to acknowledge the real challenge that we are posing to teachers, that this may be hard to do and there is admittedly work that needs to be done. If we are to support teachers – as well as students – there is a need for high quality teaching and learning materials that are accessible and inclusive, for professional development that helps teachers understand and assess student learning. Ultimately, however, NCTM needs to convey that our responsibility as educators is to "all students" and to create classrooms and communities in which "all students" truly learn and achieve.

Specific Suggestions for Revision and Comments on the Chapters

Chapter 1

p. 16, lines 4-11. It is important not to insist on mastery of basic skills before teachers move on with the curriculum--there are multiple pathways to learning math and students may grasp higher-order ideas and skills without having mastered certain basic skills (see comments on Chapter 2, p. 28 for more details)

p. 17, line 18 add: as well as those students who have traditionally not been expected to be so inclined.

Chapter 2

p.23, line 13 add: with tracking and "pull out" programs, in which students, based on perceived academic, physical and mental ability and disability, are sorted into different instructional sequences or replacement programs that often result in inequitable educational opportunities and outcomes for students.

p. 23, line 31 add: students with and without disabilities.

p.24, line 25 change: Equity will be achieved when excellent mathematics instructional programs exist that aim to achieve excellent outcomes for every student in every school.

p. 24, line 38 add: Females, students who live in poverty, students with disabilities and/or students receiving special education services, students who are not native speakers of English....

p. 24, line 40 change “victims of low expectations” to “targets of low expectation”.

p. 25, line 3 add: ...in the grouping of students for instruction, in labeling students, and in differential assignments.

p. 25, line 14 add: Although gender inequity still exists in some schools and mathematics classrooms, and may be more pronounced among some subgroup of girls than others, it is clear that overall the “gender gap” has been decreasing.

p. 25, line 29 add: ...ethnicity, poverty, and disability.

p. 26 , line3 add: Low expectations have traditionally been one of the greatest barriers for students with disabilities especially in math. Often, students with disabilities, and in particular students with learning and cognitive disabilities, have not been expected to do well in math. For instance, students with learning disabilities are still pulled out of mainstream math classes to receive resource room services in other curriculum areas such as reading and writing that are thought to be more important. As another example, the term “learning disability” is often thought of as being synonymous with not being able to do math. This erroneous assumption reveals itself when students who do not do well in math are labeled as learning disabled, and students who have been diagnosed as learning disabled are not expected to do well in math.

p.26, line 9-16 add and revise: Outcome data that are disaggregated by a multiplicity of demographic variables such as ethnicity, language, gender, disability, or social class and are interrelated with variables that serve as indicators for the educational process employed can be very useful in establishing not only differential achievement patterns, but also in identifying what kinds of district policies and classroom practices work for whom.

p. 26, line14-16 add and revise: States, provinces, school districts, schools, and classroom teachers all should examine the extend to which mathematics instructional programs produce equitable outcomes and thus help to insure that outcomes are not predictable based on factors such as gender, race, class, disability or language.

p. 26, line 35 add: ..., learning materials and technology in accessible formats, and after-school programs. Some students may need the recognition and appreciation of alternative, creative thinking and problem-solving strategies that they may bring to mathematics. Others...

p.26, line 37 add: ...other than English, and students with disabilities that affect language processing and use, may....

p.27, line 3 add: ...and new mathematical ideas; make materials more accessible to students with disabilities and help to foster their full participation;

p. 28, line 12 -21 The need for teaching mathematics concepts in such a way that they always build on one another needs to be reconsidered. The unique strengths that children with disabilities bring to the math classroom may make the traditional linear or sequential order of math education inappropriate. While a child with a disability may have difficulty with certain mathematical concepts that are considered basic, he or she may be able to master more advanced concepts even without some of the basic skills, and the mastery of more advanced concepts, in turn, may then facilitate the mastery of basic skills.

The following case, quote from one of our reviewers, who is the parent of a 12-year old girl diagnosed with autism, helps to illustrate this point:

"My daughter could not grasp basic addition or subtraction for several years. Nor could she understand that a higher number represented more of, or a greater amount of something, than a smaller number. No matter how many M&M piles we worked with, or how many number lines we posted around the house, she simply couldn't get it. The schools reaction to this was to simply continue to push these concepts until she was able to memorize some simple addition, but still had no understanding. She could not progress to other topics, and was behind the other students. This continued until fourth grade, when her new teacher decided to put calculations aside and start my daughter on fractions. I thought she was crazy. However, my daughter grasped the concept of fractions in one day, more quickly than the entire class. You see, while my daughter is considered speech impaired, she is uniquely talented visually. She embraced the "pie" diagrams of fractions, and immediately began comparing fractions successfully. She has also since begun studying basic geometry concepts and loves it! Interestingly, her success with these topics has led to greater success and confidence in computation."

To insist that students master the basics before they move on to more advanced concepts may leave disabled students stuck in basic skill drill and practice mode year after year and deprive them of finding their way into math building on their unique talents.

The Principles and Standards document needs to recognize that there could be multiple pathways into math, and discuss what can be done if a student doesn't master a fundamental or basic concept.

p.28, line 25 add: All citizens...

p. 28, line 35 This is a place where an example illustrating how teachers can connect mathematical ideas to the experience and understanding of disabled students would be helpful.

p.32, line 6 add: Teachers need to pay attention to such things as the frequency with which they call on diverse groups of students, the wait time between question and student response, the nature of their own response to students' responses, in order to create a safe environment that fosters thinking and risk.

p.35, line10-21 It is important to recognize here that conceptual and procedural knowledge may connect in different ways for different students. Some students may bring talents in conceptual knowledge, and others in procedural knowledge. As one of our reviewers put it:

"For example, many children with my daughter's diagnosis of autism are considered human "calculators." Even my daughter, who struggles with 2+2, can calculate anyone's

Chinese Zodiac sign when given their birth year by knowing that it repeats every twelve years. We still don't know how she does this, but it is a talent that should be seen as an asset."

p. 36, line 3 add: These techniques are essential for all students, including those with disabilities.

p. 37, line 4 add: ...decision-making that are equally valid for students from different demographic groups, without unnecessary...

p. 37, line 27 add: ...guidance. For students who are classified as disabled the Individualized Education Plan (IEP) includes specific long and short-term goals and serves as an additional source of guidance for setting specific learning goals.

p. 38, line 23 add: The use of a variety of different assessment methods that tap into a variety of different learning styles will provide teachers with multiple windows on students' performance and will help them to recognize the varied talents that students bring to the math classroom. Since all students with disabilities are now required to participate in standards-based assessments, assessment tools need to be available in multiple formats to insure that students with disabilities will have access to them.

p.38, line 35 add: Students who lack fluency in the language of instruction or who have a disability that affects their language processing or use may also know...

p. 40, line 40. Add - For students with disabilities, technology can make it possible to participate in the mathematics classroom and offer alternative routes to approaching and understanding mathematical concepts. Materials should be in multiple and accessible formats (disk, audio, braille for example). The Equal Access to Software Project points out that the mathematic and scientific notation, graphs, charts, drawings and three-dimensional models that are prevalent in the science, engineering and mathematics fields are often the hardest for people with disabilities to access, especially those who are blind or visually impaired. There is software, however, that provides tactile visual simulation, is helpful not only to blind students but provides all students with another way to understand, for example, dimensions and proportional relationships in a three-dimensional object or the creation and meaning of a two-dimensional matrix.

p. 41, line 7 add: ...in physics classes. Assistive technology can make it possible for blind and visually impaired students to gain access to such visualizations.

p. 41, line12 add: Computers and other technologies can give students a means of experiencing interactive lessons that go beyond what they can otherwise experience. For example, micro-based laboratories make it possible for a blind student to read a thermometer that might otherwise require sight. Dynamic geometry programs can permit a student with a physical disability to investigate ideas that other students might explore on a rubber-band board. The door is now open for students to experience some of the wonder of modern mathematics, but only if all students have access to the necessary technology.

p. 43, line39 add: However, physical access to the technology alone is not sufficient to insure equity. Students bring different attitudes and life experiences to the use of technology that teachers need to keep in mind to prevent technology from being part of the big divide. It is essential for teachers to ensure that the *use* of technology within the classroom is equitable. For instance, they need to ensure that girls, as well as boys, take time on the computer, and that students with disabilities are as actively engaged on computers as their classmates. Educators also need to insure that technology is accessible for students with disabilities. Computer accessibility for these students means that all information and controls can be read and manipulated and that all output is perceivable by the user. The dynamic geometry programs that bring such interesting options to some students will probably cause difficulty for students with visual impairments unless tactile printouts are available. The technology to make this possible is currently emerging on the market. Video-based measurement investigations are also an enriching addition to the curriculum, but might exclude deaf students if closed captions are not provided. Captioning technology for digital video is available now.

p. 43, line 39 change: To deprive students of such opportunities is to limit them, and that is fundamentally inequitable.

Chapter 3

Chapter 3 is clear and reflects an effort to include strategies that have been validated by research. That is one of the difficulties we have in recommending additions to this chapter, since as noted in the general comments, there is a dearth of validated curriculum related to students with disabilities with the exception of the SAVI/SELPH work out of the University of California that might be useful as an example in the measurement section. The writers may also consider including more on assistive technology in the section on tools for math. And although there is little discussion in the text about accommodations for students with disabilities (and it probably is not appropriate to insert them in this section), the section includes a number of examples that would certainly help students with particular disabilities to understand math better; they just aren't presented explicitly. For example, there are references to multi-sensory learning (using more than one sense to get a concept across – hearing alternating notes on a xylophone and seeing alternating colors in a banner) and student-generated strategies, particularly in the geometry and spatial sense section, and taste in the problem-solving section, that could be highlighted as also benefiting students with disabilities.

Chapter 5

One of our teacher reviewers, who works primarily with students with learning disabilities, did a careful analysis of each of the standards in relation to what might be difficult or easy for her students. One of her themes is that her students often understand more than they are capable of demonstrating and that it is our job to figure out how to make those opportunities for expressing their knowledge more available. Again, this applies to students with disabilities as well.

There needs to be "a clear statement that NCTM recognizes that not all children acquire skills and concepts at the same rate. Page 151, line 16-17 states "that students in grades 3-5 must also understand and bear responsibility for their learning." For children with special learning needs to take on that responsibility effectively, "the learning environment, the tools, materials, and the curriculum [must be] ...appropriate to their learning needs."

Standard 1: Number and Operations.

This is the area where our reviewers disagreed, with one teacher feeling strongly that many of the learning disabled students she teachers enter third grade without having a clear sense of counting, one-to-one correspondence, place value, or operations of addition and subtracting, and moving them on to multiplication and division without these basics is a serious mistake.

> In my opinion, these children must be given more experience to master these concepts, to have more experience in understanding the symbolic respresentations of mathematics, and more practice in developing and refining their own constructions within the framework of problems which seem real and plausible, and which generate interest and motivation.

The parent reviewer and science educator cited earlier is clearly of a different mind and urges that the linear sequence not be adhered to rigidly, and that other routes to introducing mathematical concepts be introduced even if basic mastery is lacking. As with her daughter, it may be that the alternative route opens up access to understanding the basic number operations skills.

Fluency and the ability to recall are critical skills for this age group, but if students have difficulty with language, they may need strategies for building this fluency. "Simple strategies such as mnemonics, color coding, introduction of structured recording materials can be very useful...as well as time to play and to use the basic facts in game formats and in real problem-solving experiences."

Re estimation skills: "Estimation practice should be built into almost every lesson so that the children get to think about estimation as the first step in attacking any problem. It is my suggestion, because I feel it helps build skills of metacognition that, as they estimate, they talk about strategies they will employ for actual solution."

Standard 2: Patterns, Functions, and Algebra

Most of the comments from our reviewers on this section are applicable as well to students without disabilities, and the writers may want to consider inclusion of some of these ideas referencing their potential benefits to both disable and non-disabled students. For example:

> In my experience with learning disabled students, ...this is an area where they are able to shine. They are able to recognize and continue patterns which are numeric, alpha-numeric, or geometric. Depending on the nature of their disabilities, some kids are better able to describe the patterns, while others are better at solving and continuing the patterns...Children with visual/perceptual difficulties may need more guidance and a more structured approach to the use of tables and graphs. It is essential that these pictorial representations are free of excess clutter and visually large and simple...They may need help in learning how to read the tables and graphs, particularly those which require that they locate a point on a line graph which requires that they read location both by the abscissa and ordinate. I have noted that these children often have difficulty using a "times table." Creating an "L" shaped tool which coordinates the vertical with the horizontal enables them to find their place on the table or graph. With practice, they are often able eventually to do this by themselves.
>
> I have come to understand that some children process from the top down and others from the bottom up. It is important help the kids who process better from whole to part to play games which encourage them to "guess the rule." It is very helpful for them to complete patterns, functions or relations and then to describe them. In the act of describing they begin to find the language to explain their thinking...Other children may have come to faulty generalizations that need to be worked through. These children seem to take

one example and formulate a rule which works for that example, but they neglect to verify with other examples for generalization.

A teacher reviewer notes that "the most important statement in this unit, to me, emerges on page 179, lines 21-22: 'Measurement activities are more engaging and thought-provoking if they are carried out for a purpose.' While I believe this is true in general, children with disabilities who often struggle to accomplish, are easily frustrated if they feel they are doing busy work. They intuitively know that purposeful and meaningful work is demanding...and worth the effort...It is extremely important to make the activities useful and meaningful to these young learners."

Standard 5: Data Analysis, Statistics and Probability

"Most kids of all ages like this aspect of math. It appeals because they see its usefulness all around them...While the disabled youngster may have some difficulty with the organization and representation of data, they are often fascinated by the concepts embedded in the strands – can our knowledge of probability help us to 'beat the odds.' This section of the curriculum can often be used in the service of reinforcing general computational skills as well...Kids who have had only a very shaky understanding of basic fractions seem to get a much clearer picture of hte meaning of htose possibiliteis which exist between 0 or no chance an event will occur, and 1, the event will always occur."

Standard 6: Problem Solving

The teacher of students with learning disabilities notes that "Of all the areas in the curriculum, this is certainly the most essential and often the most difficult for children with learning disabilities...Perhaps the most important issue in math education for the disabled child is how best to help them become more effective problem-solvers. The ways in which we help these children to develop strategies to untangle language, to make appropriate translations, to organize and systematize their thinking and to incorporate their skills in estimation and logic will undoubtedly help them to become better problem solvers."

There are some research-based strategies that the Department of Education has described that can be of help here (See Explicit instruction and cognitive strategy instruction, which basically help students to understand the "tricks of the trade" that their peers might have intuited along the way, and which break down the processes of solving problems into manageable and coherent parts.)

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Appendix A

Reviewers

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